Class - 11 Chapter - 2

Relations and Functions

Exercise 2.1 - Basics

Sets.
↓
ordered Pairs
↓
C.P.
↓
Rel.
↓
Fn.

Ordered Pairs : (x, y)

2 elements in Particular order

(x, y) $(2, 3)$ $(5, -3)$ $(3, 2)$

$(2,3) \neq (3,2)$

$x \in Set_1$

$y \in set_2$

set $= \{2, 3\} = \{3, 2\}$

Prop. of Ordered Pairs.

(I) $(x, y) = (a, b)$ $\Rightarrow$ $x = a$, $y = b$

ex. If $(x+3, y-1) = (7, 8)$, then find

$x \longrightarrow x+3 = 7 \Rightarrow x = 4$

$y \longrightarrow y-1 = 8 \Rightarrow y = 9$

Cartesian Product of Sets $(A \& B) = A \times B$

$\neq B \times A$

A = set (nonempty)

B = set (non empty)

$A \times B$ = Set of all possible ordered Pairs from A to B.

$= \{(x, y) : x \in A \text{ and } y \in B\}$

$B \times A$ = Set of all possible ordered Pairs from B to A

$B \times A = \{(x, y) : x \in B, y \in A\}$

Examples.

$A = \{$ Red, yellow $\}$ ← Colors

$B = \{$ Pants, Shirt $\}$ ← Apprals

$A \times B = \{$(Color, App.), , $\}$

↙ this type

$A \times B = \{$(Red, Pants), (red, Shirt), (yellow, pant), (yellow, Shirt)$\}$

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eg:

$A = \{1, 2, 3\}$

$B = \{a, b, c\}$

	a	b	c
1 →	(1,a)	(1,b)	(1,c)
2	(2,a)	(2,b)	(2,c)
3	(3,a)	(3,b)	(3,c)

↑

$A \times B$

$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a), (3,b), (3,c)\}$

$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3)\}$

$B \times A = \{(x, y) : \underline{x \in B}, y \in A\}$

$x \to$ a, b, c $y \to$ 1, 2, 3

In above example

$n(A) = 3$

$n(B) = 3$

$n(A \times B) = 9 = n(B \times A)$

Properties of ~~orded~~ ordered Pairs.

(I) $n(A) = n_1$, $n(B) = n_2$ $\quad A \times B \neq B \times A$

$$\boxed{n(A \times B) = n(B \times A) = \text{no. of elements} = n_1 \cdot n_2}$$

(II) If Either A or B is empty set then $A \times B = \phi = \{ \}$

$A = \{ \}$ $B = \{1, 2, 3\}$

$\times$ (, 1) (, 2) (, 3)

Doesnot make any sense.

(III) If either A or B is infinite set then $A \times B$ is also an infinite set.

$A = \{2\}$ $B = \{1, 2, 3 \ldots\}$

$A \times B = \{(2,1), (2,2), (2,3), (2,4), (2,5) \ldots\}$

$A \to 2$, $B \to 1, 2, 3 \ldots$

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(IV) $A \times A = \{(x, y),\ x \in A,\ y \in A\}$

e.g. $A = \{1, 2\} \longrightarrow n(A) = 2$

$A = \{1, 2\} \longrightarrow n(B) = 2$ $\quad \underline{n(A \times B) = 4}$

$A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$

(V) $A \times A \times A$ = set of ordered triplets

$= \{(x, y, z) : x \in A, y \in A, z \in A\}$

e.g. ~~$A = \{(a, b)\}$~~ $A = \{a, b\}$

$A \times A \times A = \{(a, a, a), (a, a, b), (a, b, a), (a, b, b), (b, a, a), (b, a, b), (b, b, a), (b, b, b)\}$

$\begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \rightarrow (a, a, b)$

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(IV) $A \times A = \{(x, y), x \in A, y \in A\}$

e.g. $A = \{1, 2\} \longrightarrow n(A) = 2$

$A = \{1, 2\} \longrightarrow n(B) = 2$ $\quad \underline{n(A \times B) = 4}$

$A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$

(V) $A \times A \times A$ = set of ordered triplets

$= \{(x, y, z) : x \in A, y \in A, z \in A\}$

e.g. ~~$A = \{(a,b)\}$~~ $A = \{a, b\}$

$A \times A \times A = \{(a,a,a), (a,a,b), (a,b,a), (a,b,b), (b,a,a), (b,a,b), (b,b,a), (b,b,b)\}$

(a,a,b)

Exercise 2.1

(1) $\left(\frac{x}{3}+1, y-\frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$

$\frac{x}{3}+1 = \frac{5}{3} \Rightarrow \frac{x}{3} = \frac{5}{3}-1 = \frac{2}{3} \Rightarrow x = 2$

$y-\frac{2}{3} = \frac{1}{3} \Rightarrow y = \frac{2}{3}+\frac{1}{3} \Rightarrow y = 1$

(2) $n(A \times B) = n(A) \times n(B)$

$= 3 \times 3 = 9$ elements

(3) $G = \{7, 8\}$ $\quad H = \{5, 4, 2\}$

$G \times H = \{(7,5), (7,4), (7,2), (8,5), (8,4), (8,2)\}$

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$H \times G = \{(5,7), (4,7), (2,7), (5,8), (4,8), (2,8)\}$

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(4) ~~A~~ $P = \{m, n\}$

(I) F $Q = \{n, m\} = \{m, n\}$

$P \times Q = \{(m,n), (m,m), (n,n), (n,m)\}$

(II) T

(III) $LHS = A \times (B \cap \phi)$

$\{3,4\} \cap \{\}$

$= A \times (\phi)$

Empty

(T) $= \phi$

$= RHS$

(5) $A = \{-1, 1\}$

ordered triplet

$A \times A \times A = \{(-1,-1,-1), (-1,-1,1),$
$(-1,1,-1), (-1,1,1)$
$(1,-1,-1), (1,-1,1)$
$(1,1,-1), (1,1,1)\}$

(6) $A \times B = \{(a,x), (a,y), (b,x), (b,y)\}$

~~A = {~~ $A = ? = \{a, b\}$

$B = ? = \{x, y\}$

(7) $A = \{1,2\}$ $B = \{1,2,3,4\}$

$C = \{5,6\}$ $D = \{5,6,7,8\}$

(i) $\boxed{A \times (B \cap C) = (A \times B) \cap (A \times C)}$

$B \cap C = \phi$

$LHS = A \times (B \cap C)$

$= A \times (\phi)$

$= A \times \phi = \phi$

~~RHS = ...~~

$A \times B = \{(1,1)(1,2), (1,3)(1,4), (2,1)(2,2)(2,3)(2,4)\}$

$A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

$RHS = (A \times B) \cap (A \times C)$

$= \phi = LHS.$ ✓

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(ii) $A \times C$ is a subset of $B \times D$

Prove.

$A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

$B \times D = \{(1,5), (1,6), (2,5), (2,6), (1,7), (1,8), (2,7), (2,8), (3,5) \ldots\}$.

Clearly $A \times C \subset B \times D$

---

(8) $A = \{1,2\}$

$B = \{3,4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

no. of elements in $A \times B = 4 = n$

no. of subset of $A \times B = 2^n = 2^4$

$= 16$

All subsets of $A \times B \Rightarrow$

ϕ

$\{(1,3)\}$

$\{(1,4)\}$

$\{(2,3)\}$

$\{(2,4)\}$

$\{(1,3), (1,4)\}$

$\{(1,3), (2,3)\}$

$\{(1,3), (2,4)\}$

$\{(1,4), (2,3)\}$

$\{(1,4), (2,4)\}$

$\{(2,3), (2,4)\}$

$\{(1,3), (1,4), (2,4)\}$

$\{(1,3), (1,4), (2,3)\}$

$\{(1,3), (2,3), (2,4)\}$

$\{(1,4), (2,3), (2,4)\}$

$\{(1,3), (1,4), (2,3), (2,4)\}$

16

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(9) $n(A) = 3$.

$n(B) = 2$

$(x, 1), (y, 2), (z, 1) \in A \times B$

$A = \{x, y, z\}$ ✓

$B = \{1, 2\}$ ✓

(10) no. of elements in $A = n = ?$

$n(A \times A) = n \times n = n^2 = 9$

$\Rightarrow n^2 = 9$

$n = \pm 3$

$\boxed{n = 3}$

no. of elements in $A = n = 3$

$(-1, 0), (0, 1) \in A \times A$

$A = \{-1, 0, 1\}$ ✓

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Basics of Exercise $\boxed{2.2}$

Relations and Functions. — Chapter 2

Relation:

A relation (R) from a non-empty set A to a non-empty set B is a subset of cartesian product $A \times B$.

Sets.
↓
ordered Pairs
↓
C.P. $A \times B$
↓
Rel^n.
↓
Fn^n.

Set Builder Form:

$R = \{ (x,y) : \text{Relation (equation) in } x \;\&\; y \}$

$x \in A, y \in B$

↓

R from A to B / ARB / xRy

$\boxed{R : A \to B}$

Here image of $x = y$

Preimage of $y = x$

Example.

Set $A = \{$ Amit B., ~~Aamir~~ Aamir K., Salman K., Akshay K.$\}$

↑
actors.

Set $B = \{$ Baghban, PK, Singham, 3 I $\}$

Cartesian product $(A \to B)$ $\boxed{4 \times 4 = 16}$

$A \times B = \{ \text{All ordered pairs} \}$
↓
total 16.

$R_1 = \{ (x,y) : \text{'x' acted in movie 'y'} \}$

$\boxed{R_1 : A \to B}$

↑
Set Builder

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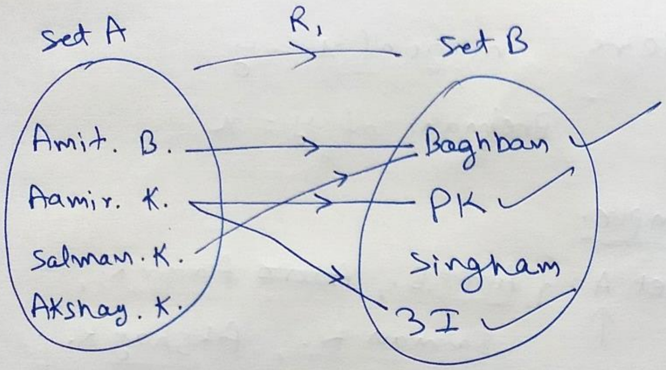

Arrow Diagram.

$R_1 : A \to B$

Roster form:

$R_1 = \{$ (AB, Baghban), (Aa.K., PK), (Aa.K., 3I), (Salman.K., Baghban) $\}$

image of AB $=$ Baghban
image of Aa.K. $=$ PK, 3I

$R_1 : X \to Y$

Pre-image of Baghban = Amit B., Salman K.

Pre-image of PK = Aamir K.

Domain: set of first elements (connected) in any Relation.

Domain = { AB, Aa.K, Salman.K }

Range: Set of second elements (connected) in any relation.

Range = { Baghban, PK, 3I }.

Codomain: the complete set B

$R_1 : A \to B$ → the second set.

Codomain = { Baghban, PK, Singham, 3I }

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Note: Range $\subseteq$ Codomain

Range ↓ set of only connected elements of B

Codomain ↓ Complete set B

Note: R from A to B : $R: A \to B$

$R: A \to A$ = R from A to A

= R on A

Example: $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : x = y + 2\}$ defined on A.

$x \in A$ $\quad y \in A$ $\leftarrow$ $R: A \to A$

$x = y + 2$ $\quad y \in A$,

$y = 1$ $\quad x = 3 \in A$ ✓

$y = 2$ $\quad x = 4 \in A$ ✓

$y = 3$, $x = 5 \in A$ ✓

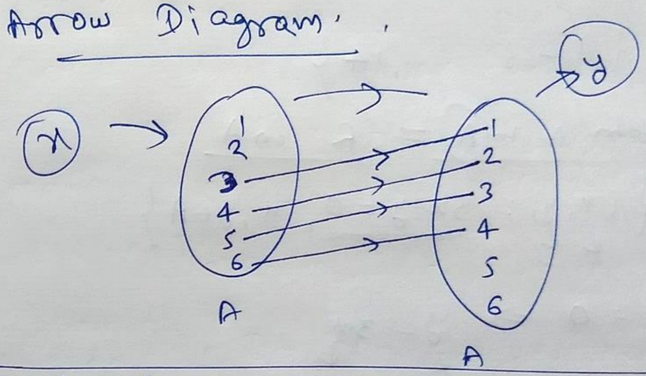
$y = 4$, $x = 6 \in A$ ✓

$y = 5$, $x = 7 \notin A$

$R = \{(3, 1), (4, 2), (5, 3), (6, 4)\}$

Roster form ↑

Arrow Diagram:



\# No. of Relation from A to B = no. of subsets of Cartesian product $A \times B$

$= 2^{p.q}$

$n(A) = p$

$n(B) = q$

$n(A \times B) = pq$

2^n

E.g. $A = \{2, 3, 4\}$

$B = \{a, b\}$

No. of relations from A to B

$= 2^{3 \times 2} = 2^6 = 64$

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Exercise : 2.2 (All questions)

(1) $A = \{1, 2, 3, \ldots, 14\}$

R from A to A = R on A

$R = \{(x, y) : 3x - y = 0,\ x, y \in A\}$

$3x - y = 0 \Rightarrow y = 3x$

$y \in A$ $\quad$ $x \in A$

x	1	2	3	4	$\cancel{5}$
$y = 3x$	3	6	9	12	$\cancel{15}$

→ not possible.

$R = \{(1,3), (2,6), (3,9), (4,12)\}$.

Domain $= \{1, 2, 3, 4\}$

Range $= \{3, 6, 9, 12\}$

Codomain $= \{1, 2, 3, \ldots 14\}$
$\parallel$
A

(2) $R : N \to N$ = R on N

$R = \{(x, y) : y = x + 5\}$

$x \in N$ $\quad$ x is less than 4

$x = 1, 2, 3$

$y = 6, 7, 8$

$R = \{(1,6), (2,7), (3,8)\}$

Domain $= \{1, 2, 3\}$

Range $= \{6, 7, 8\}$

(3) $A = \{1, 2, 3, 5\}$

$B = \{4, 6, 9\}$

Diff. = odd = odd − Even

Even − odd

$R = \{(1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)\}$

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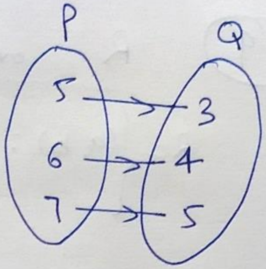
④

(i) Set Builder Form:

$R = \{(x, y) : x - 2 = y,\ x \in P,\ y \in Q\}$



(ii) Roster Form:

$R = \{(5,3), (6,4), (7,5)\}$

Domain $= \{5, 6, 7\}$

Range $= \{3, 4, 5\}$

⑤ $A = \{1, 2, 3, 4, 6\}$

$\frac{b}{a}$

$R = \{(a, b) : a, b \in A,\ b \text{ is exactly divisible by } a\}$

$\frac{b}{a}$ $\frac{b}{1}$ $\frac{2}{1}$ ✓ $\cancel{\frac{6}{4}}$ $\frac{-}{2}$ $\cancel{\frac{6}{3}}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)\}$

Domain $= \{1, 2, 3, 4, 6\}$

Range $= \{1, 2, 3, 4, 6\}$

⑥ $R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

$R = \{(0,5), (1,6), (2,7), (3,8), (4,9), (5,10)\}$

Domain $= \{0, 1, 2, 3, 4, 5\}$

Range $= \{5, 6, 7, 8, 9, 10\}$

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(7) $R = \{(x, x^3) : x \text{ is prime no. less than } 10\}$

$x = 2, 3, 5, 7,$

$R = \{(2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3)\}$

$= \{(2,8), (3, 27), (5,125), (7, 343)\}$

(8) $A = \{x, y, z\} \longrightarrow n(A) = 3 = p$

$B = \{1, 2\} \longrightarrow n(B) = 2 = q$

no. of relations from A to B

$= 2^{p.q}$

$= 2^{3\times 2}$

$= 2^6 = 64$

(9) R on Z

$R = \{(a, b) : a, b \in Z, a-b \text{ is an integer}\}$

set of integers

$\boxed{\text{Int}_1 - \text{Int}_2 = \text{Int.}}$

$a \in Z$

$b \in Z$

Domain = Integers = Z

Range = Z

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Class - 11 Maths

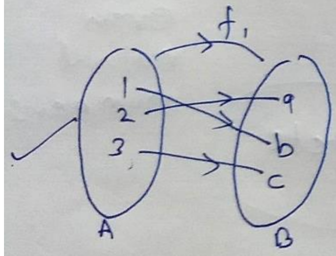
Sets
↓
ordered Pairs
↓
$A \times B$
↓
Rel^n
↓
Fn^n

Functions | Map | Mappings:

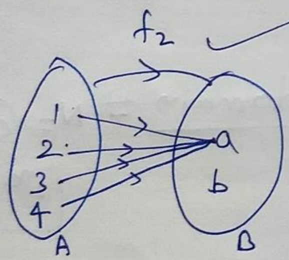
Definition (I): $\boxed{f : A \to B}$

A relation 'f' from a set 'A' to a set 'B' is known as function if each element of 'A' has unique (one and only one) image in 'B'.

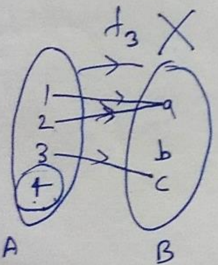
f_1



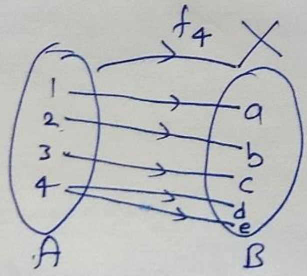
f_2



f_3 ✗



f_4 ✗



✓ $f_1 = \{(1,b), (2,a), (3,c)\}$

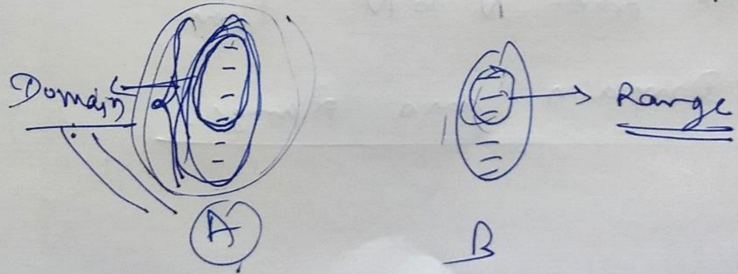
✓ $f_2 = \{(1,a), (2,a), (3,a), (4,a)\}$

✗ $f_3 = \{(1,a), (2,a), (3,c)\}$ $A = \{1,2,3,4\}$
$\cancel{(4, \ )}$ $B = \{a,b,c\}$

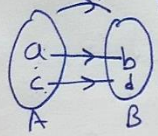
✗ $f_4 = \{(1,a), (2,b), (3,c), (4,d), (4,e)\}$

Definition - (II)

In other words, a function f from A to B such that the domain of 'f' is $\boxed{A}$ and no two distinct ordered pairs in f have the same first element.



Note: $f : A \to B$



$f = \{(a,b), (c,d)\}$

(I) If 'f' is a function from A to B and $(a, b) \in f$

then $\boxed{f(a) = b}$

~~$P(x) = x^2 + 3$~~
~~$P(1) = 1^2 + 3$~~

image of 'a' under 'f' = b

image of 'c' under 'f' = d

Preimage of 'b' under 'f' = 'a'

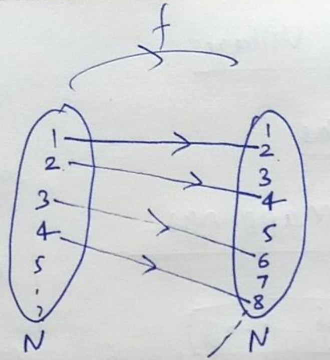
Preimage of 'd' under 'f' = c

---

e.g. fun^n $f = \{(x, y) : y = 2x, \; x, y \in N\}$ (on N)

defined from N to N then

find Domain and Range:



x $\quad$ $\boxed{y = 2x}$ ☆

Domain = $\{1, 2, 3, 4, 5, 6, \ldots\} = N$

Range = $\{2, 4, 6, 8, \ldots\}$ = set of even natural numbers.

Codomain = N = Complete 2nd set

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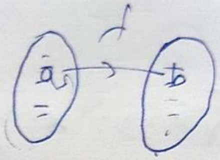
Real valued function :



$f(a) = b$

input → Output → value

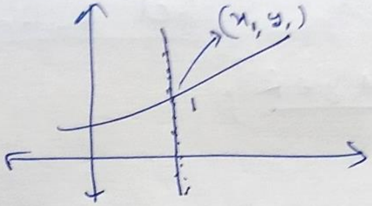
those functions whose range is 'R' (real numbers) or subset of 'R'.

Real Function:

those functions whose domain and range both are set of real numbers or subset of real numbers.

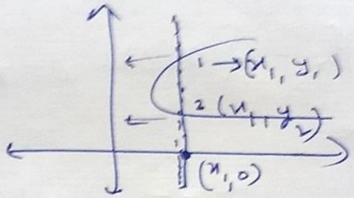
Graphical Definition of Function

If vertical line does not cut the graph of relation 'f' for more than one point then 'f' is known as function.



f_1 ✓

Fn.



f_2 ✗

not a Fn.

Set Builder Form.

interval

$f_1 = \{(x, y) : y = x^2,\ \cancel{x\in}\ x, y \in [0, \infty)\}$ → Yes/no

$f_2 = \{(x, y) : x = y^2,\ x, y \in [0, \infty)\}$ → Yes/No

$f_1 = \{(0,0),$

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Set Builder

$f_1 = \{(x, y) : y = x^2,\ x, y \in R\}$

$f_2 = \{(x, y) : x = y^2,\ x, y \in R\}$

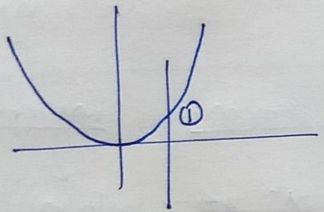
$y = x^2$

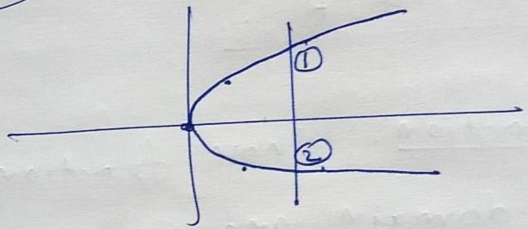
x	1	2	-1	-2
y	1	4	1	4

$x = y^2$

$x = (-1)^2 = 1$

x	0	1	1	4	4
y	0	1	-1	-2	2





Fn ✓

Not a Fn ✗

$y = x^2$ $\{(0,0), (1,1), (2,4), (-1,1), (-2,4)$ - - - - $\}$

(x, y)

$\uparrow\ \uparrow$

$-1\ \ 1$

$x = y^2$

(x, y)

$f_2 = \{(0,0), (1, 1)$

$(1, -1)$, - - -

Before Exercise 2.3

Inequality   Domain   Range

$\boxed{\text{Most important video lecture}}$

Inequalities $(>, <, \geq, \leq)$

$5>3$ ✓, $-2<-1$ ✓, $0 \leq 7$ ✓, $x>3$ ✓, $7y^2-5<100$ ✓

(I) let $a>b$

$\Rightarrow -a<-b$

⊖ multiply
⇓
Sign of inequality change

E.g. $7<10$

$-7>-10$

(II) let $a>b$

$\Rightarrow \boxed{Ka>Kb}$

+K multiply
⇓
Sign of inequ does not change.

(III) Reciprocal

$x<a<b$

Both limits required (Same sign)

$\overset{\oplus}{x}<a<\overset{\oplus}{b}$

$\Rightarrow \frac{1}{x}>\frac{1}{a}>\frac{1}{b}$

$\overset{\ominus}{c}<d<\overset{\ominus}{e}$

$\Rightarrow \frac{1}{c}>\frac{1}{d}>\frac{1}{e}$

$\overset{\ominus}{c}<a<\overset{\oplus}{b}$

⇓

Please do not take reciprocal.

$-$ 0 $+$

$\frac{1}{0}=\infty=$ Not defined

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For e.g.

Solve. (I) $3x-7 < 2$

(II) $2 < 3-4x \leq 10$

(III) $\frac{1}{2} < \frac{1}{2-3x}$

(I) $3x-7 < 2$

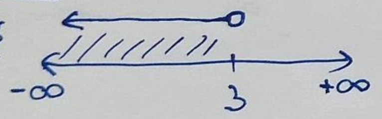
$\Rightarrow 3x < 9$

$\Rightarrow \frac{3x}{3} < \frac{9}{3}$ $(+3)$ → +ve ↓ No sign change

$\leq \Rightarrow \leq$

$\Rightarrow \boxed{x < 3}$

$\cancel{x \leq 3}$



$x \in (-\infty, 3)$

↑ small bracket ↑ small bracket (Exclude)

(II) $2 < \boxed{3-4x} \leq 10$

$\Rightarrow 2-3 < -4x \leq 10-3$

$\Rightarrow -1 < \boxed{-4x} \leq 7$

$\Rightarrow \frac{-1}{-4} > \frac{-4x}{-4} \geq \frac{7}{-4}$

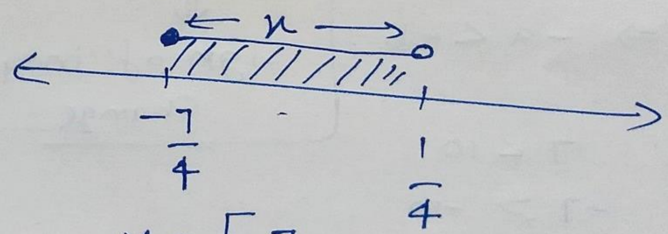
(−4 ⇒ −ve Sign change Inequality का)

$\Rightarrow \frac{1}{4} > x \geq -\frac{7}{4}$

$\Rightarrow -\frac{7}{4} \leq x < \frac{1}{4}$

$2 > 1$

$1 < 2$



$x \in \left[-\frac{7}{4}, \frac{1}{4}\right)$

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Quadratic Inequality : (Higher Degree Inequality)

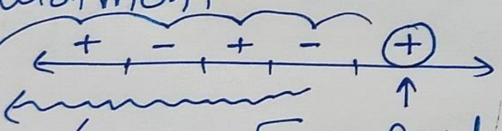
Steps. :→

(I) Make leading coefficient positive (with other side zero)

(II) Factorize

(III) Make number line & mark roots on it

(IV) Sign allotment



[then Alternate sign]

[on Right most part always (+)ve]

(V) Chose either (+)ve part or (−)ve part according to inequality

e.g. $-x^2+5x \le 6$

$\Rightarrow -(-x^2+5x) \ge -6$ (− multiply)

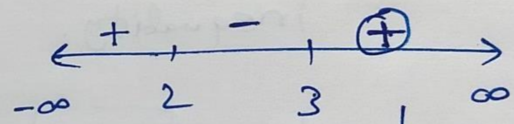
$\Rightarrow x^2-5x \ge -6$

$\Rightarrow x^2-5x+6 \ge 0$ (I step complete)

$\Rightarrow x^2-2x-3x+6 \ge 0$

$\Rightarrow x(x-2)-3(x-2) \ge 0$

$\Rightarrow \boxed{(x-2)(x-3) \ge 0}$ (II step complete)



$x=2,3$ roots.

For e.g. 10

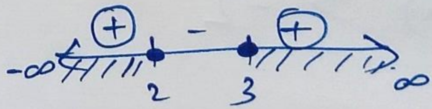
x

$(10-2)(10-3) \ge 0$

$(8\times7) = 56 \ge 0$

$(x-2)(x-3) \ge 0$

Positive (+) ⟶ choose (+) part.



Solution = $x \in (-\infty, 2] \cup [3, \infty)$

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Next example.

Solve. $\boxed{100 - x^2 > 0}$ ✓

या

$\boxed{100 > x^2}$ ✓

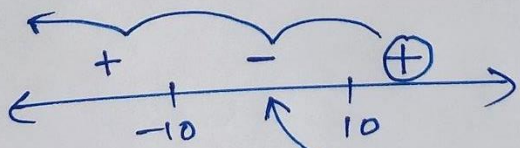
$100 - x^2 > 0$

⊖ multiply . - - -

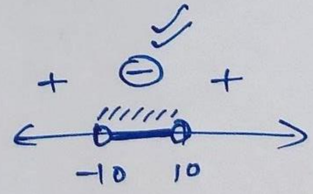
$\Rightarrow x^2 - 100 < 0$ $a^2 - b^2 = (a+b)\cdot(a-b)$

$\Rightarrow \boxed{(x+10)(x-10) < 0}$ ★

$x = -10, 10$



$\underbrace{(x+10)(x-10)}_{\ominus\text{ve Number}} < 0$



$\boxed{x \in (-10, 10)}$

Solution

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Domain

Inequality >>> Domain >>> Range

Domain = set of all possible inputs.

Range = set of all possible outputs.

b) e.g. $\boxed{f(x) = 3x - 7}$ $f(x) = y$

x → value of x ← input ← Domain(x)

$3x - 7$ → output (y)

$$y = \overbrace{f(x)}^{\text{Range(output}}$$

y → Output (Range) → Y-axis

x → input (Domain) → X-axis

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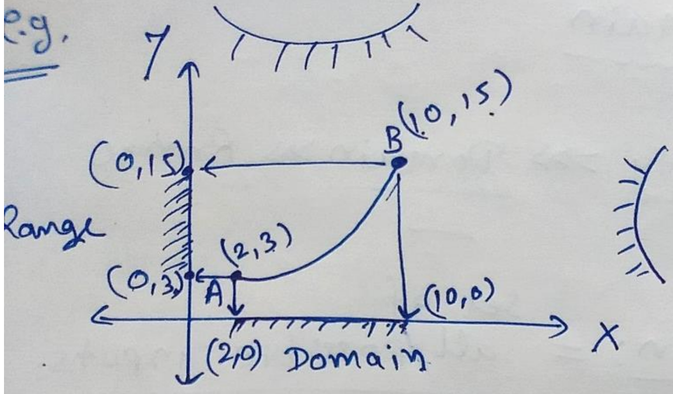
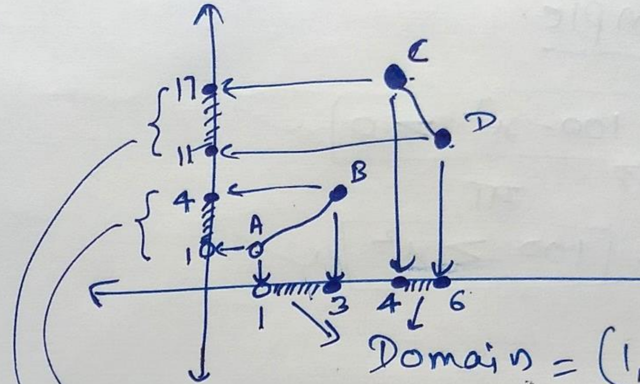
e.g.



Domain $= x \in [2,10]$

Range $= y \in [3,15]$



A(1,1)

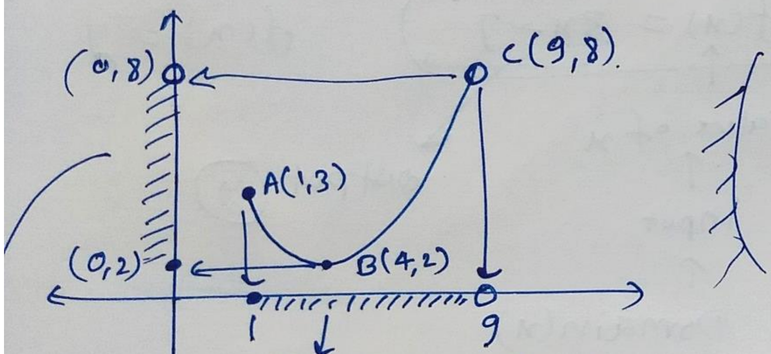
B(3,4)

C(4,17)

D(6,11)

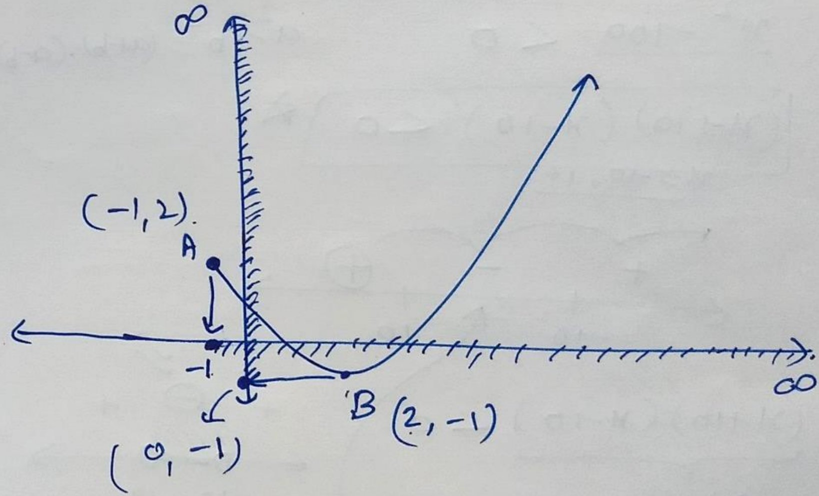
Domain $= (1,3] \cup [4,6]$

Range $= (1,4] \cup [11,17]$



Domain $= [1,9)$

Range $= [2,8)$



Domain $= [-1,\infty)$

Range $= [-1,\infty)$

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Ways to find Domain :→

$\sqrt{f(x)}$

$\Downarrow$

$\boxed{f(x) \geq 0}$

$\downarrow$

Solve,

$x \in$ - -

$\dfrac{f(x)}{g(x)}$

$\Downarrow$

$\boxed{g(x) \neq 0}$

$\downarrow$

$x \neq$ (- - -)

$\frac{f(x)}{0} = \infty$

Not Defined

$\times$ $\sqrt{\ominus}$ = imaginary No. (Not real)

$\sqrt{-1} = i =$ iota $\sqrt{-7} =$

$\sqrt{0}$ ✓ Real

$\sqrt{+}$ ✓ Real

---

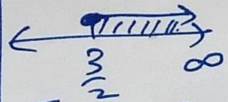
for example.

① $f(x) = \sqrt{2x-3}$

For Domain $2x - 3 \geq 0$

$\Rightarrow 2x \geq 3$

$x \geq \frac{3}{2}$



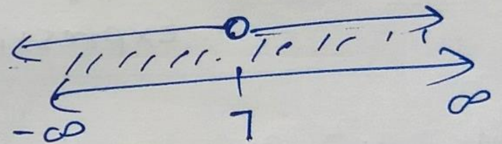
$[3/2, \infty)$

② $f(x) = \dfrac{2x+3}{x-7}$

For Domain $x - 7 \neq 0$

$\boxed{x \neq 7}$

x can be anything but not 7.



$x \in (-\infty, 7) \cup (7, \infty)$ ✓

या

$x \in (-\infty, \infty) - \{7\}$ ✓

या

$x \in R - \{7\}$ ✓

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Range: → output ✓

↓

Value of y ✓

↓

value of $f(x)$ ✓

method ≈ Start with Known Function or Expression

$y = f(x) = x^2 - 3, \quad x \in R$

Domain & Range.

Domain → $x \in R$

Domain $= R$ ✓

Range $\quad y = f(x) = x^2 - 3$

$x \in R$

Known $= x^2 =$ equal to zero or positive.

But $x^2 \neq$ negative.

Perfect square $= (ax+b)^2 \geq 0$

$\boxed{y = x^2 - 3 = f(x)}$

$x^2 \geq 0$ (Always)

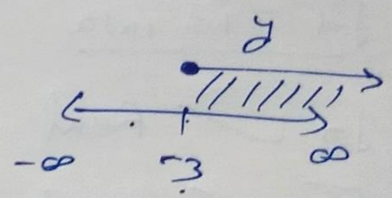
$\Rightarrow x^2 - 3 \geq 0 - 3$

$\Rightarrow x^2 - 3 \geq -3$

$f(x) \geq -3$

या

$y \geq -3$



~~y∈~~ $\quad y \in [3, \infty)$

Range $= [-3, \infty)$

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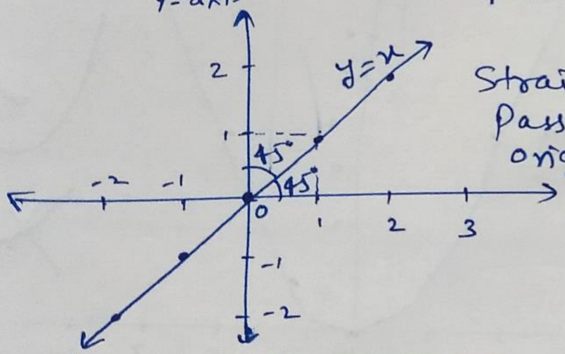
Some Functions and their Graphs

Name	Expression	Graph	☆ Domain Range

(i) Identity Function:

$f : R \to R$ defined by $\underbrace{y}_{\text{output}} = f(x) = \underbrace{x}_{\text{input}}$

output $=$ input



Straight line Passing through origin (0,0)

x	0	1	2	-1	-2
y	0	1	2	-1	-2

Domain $= R$

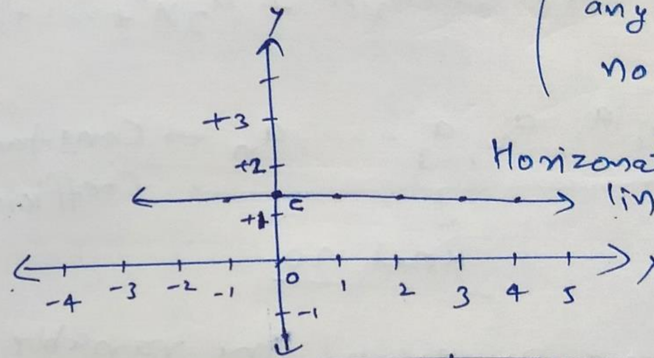
Range $= R$

Codomain. $= R$

(ii) Constant Function:

$f : R \to R$ defined by $\boxed{f(x) = y = c}$

(Constant any Real no.)



Horizonatal line

x	0	1	2	3	4	-1
y	c	c	c	c	c	c

constant output.

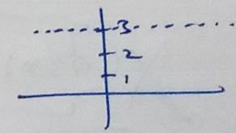
Domain $= R$.

Range $= \{c\}$ = Singleton set.

e.g. $y = f(x) = 3$

Domain $= R$

Range $= \{3\}$



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(iii) Polymonial Functions

$f : R \to R$ defined by

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n$$

$$\underbrace{a_0, a_1, a_2, a_3 \dots a_n}_{\text{Real no.}} \to \text{Constant Coefficients.}$$

n = power of the variable 'x'

= whole no. = $\{0, 1, 2, 3 \dots\}$

e.g.

$f(x) = 2x + 3$

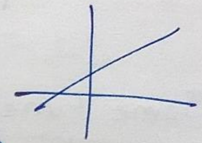
$f(x) = x$

$f(x) = 3x^3 - 5x^2 + 7x - 8$

X $f(x) = x^{2/3} - x^{100} + x^2 + 1$

$2/3 \notin W$

Note: Linear Functions



$y = \boxed{f(x) = ax + b}$, $a \neq 0$

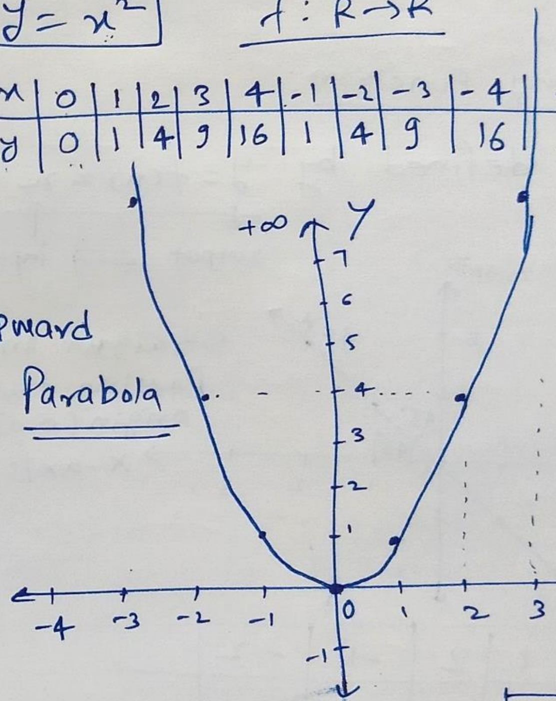
Straight line

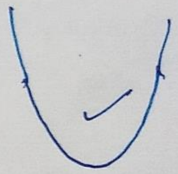
e.g.

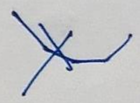
① $\boxed{y = x^2}$ $\quad f : R \to R$

x	0	1	2	3	4	-1	-2	-3	-4	...
y	0	1	4	9	16	1	4	9	16	...

Upward Parabola



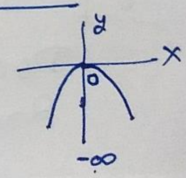




Domain = R

Range = $[0, \infty)$

$y = -x^2$



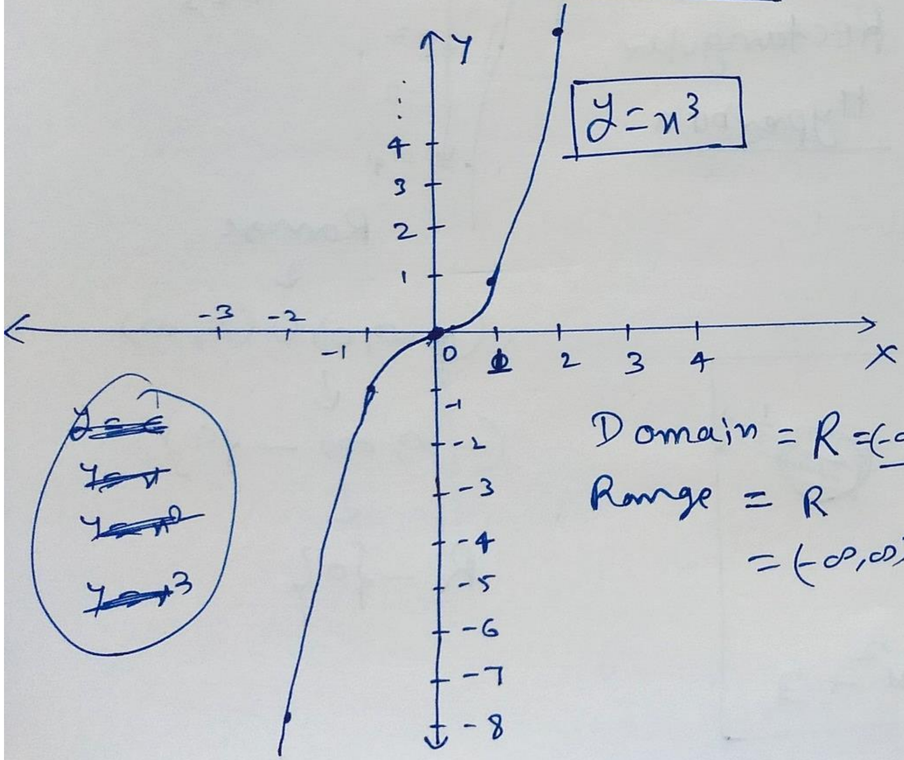
e.g. ② $y = f(x) = x^3$

$f : \underset{\uparrow \text{ input}}{R} \to R$

$\boxed{y = x^3}$

x	0	1	2	3	−1	−2	−3	. . . .
y	0	1	8	27	−1	−8	−27	

$(-1)^3 = -1 \cdot -1 \cdot -1 = -1$



Domain $= R = (-\infty, \infty)$

Range $= R$
$= (-\infty, \infty)$

(iv) Rational Functions: $= \dfrac{f(x)}{g(x)}$; $g(x) \neq 0$

$f(x), g(x) \to$ polynomial F^n.

e.g. $f(x) = \dfrac{2x+3}{3-x}$ Domain $\Rightarrow x \in R - \{3\}$

$\dfrac{x}{0} = \infty$

$3 - x \neq 0$

$x \neq 3$

e.g. $\boxed{y = \dfrac{1}{x}} = f(x)$ $f : R - \{0\} \to R - \{0\}$

$x \neq 0$

Domain $= R - \{0\}$

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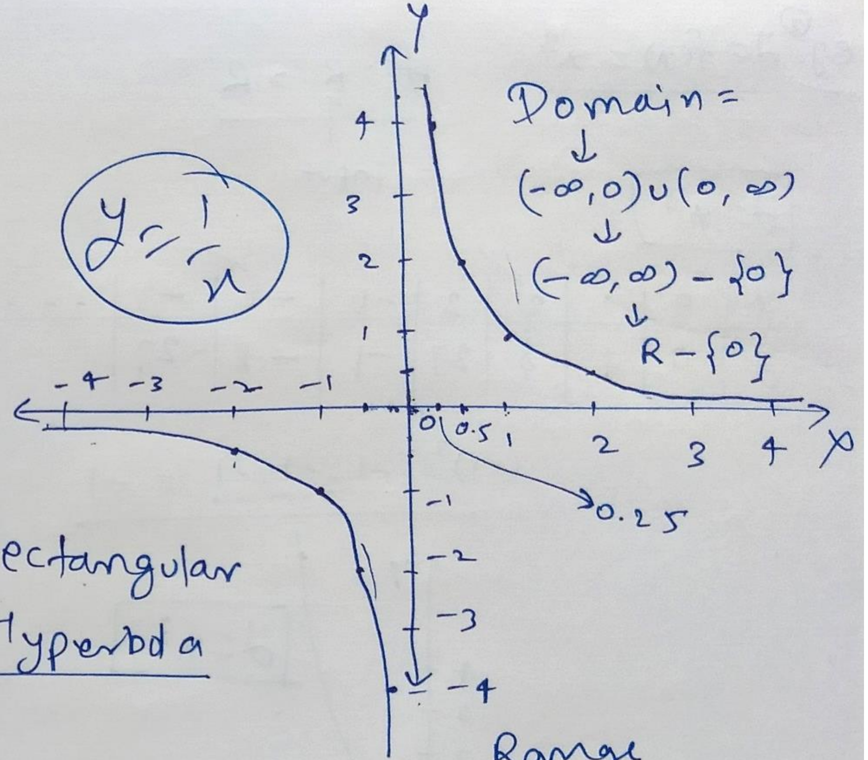
$y = \frac{1}{x} = f(x)$ $\quad$ $f : R - \{0\} \longrightarrow R - \{0\}$

x	0.25	0.5	1	2	−0.25	−0.5	−1	−2
y	4	2	1	0.5	−4	−2	−1	−0.5

$y = \frac{1}{x}$



Domain =
$\downarrow$
$(-\infty, 0) \cup (0, \infty)$
$\downarrow$
$(-\infty, \infty) - \{0\}$
$\downarrow$
$R - \{0\}$

$y = \frac{1}{x} = \frac{1}{0.25} = \frac{100}{25} = 4$

$y = \frac{1}{x} = \frac{1}{0.5} = \frac{10}{5} = 2$

$y = \frac{1}{1} = 1$

$y = \frac{1}{2} = 0.5$

Rectangular Hyperbola

Range
$\downarrow$
$(-\infty, 0) \cup (0, \infty)$
$\downarrow$
$(-\infty, \infty) - \{0\}$
$\downarrow$
$R - \{0\}$

Note: Irrational Fn.

$2, 3 \quad \frac{1}{2}, \frac{1}{3}$

$f(x) = \sqrt{x} \longrightarrow x \geq 0$

$f(x) = x^{2/3}$

$f(x) = \sqrt{(7x - 3)} + x^2 + 3$

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Some functions and their graphs:

Name	Expression	Graph	Domain Range

$$y = f(x) = |x| = \begin{cases} x, & x \geq 0 \quad \text{Positive \& zero} \\ -x, & x < 0 \quad \text{Negative} \end{cases}$$

+ (Positive) = ⊕

− (Negative) = ⊕

Ⅴ The Modulus Function

(Absolute Value Functions)

$f : R \to R$ ~~definde~~ defined by $f(x) = |x|$

e.g. $f(2) = |2| = 2$

$f(-2) = |-2| = 2 = -(-2)$

$f(0) = |0| = 0$

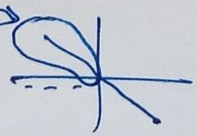
$f(-3.8) = |-3.8| = 3.8$

$x = -4$

$f(-4) = |-4| = \{ -(-4), \ -4 < 0$

$= 4 = 4$

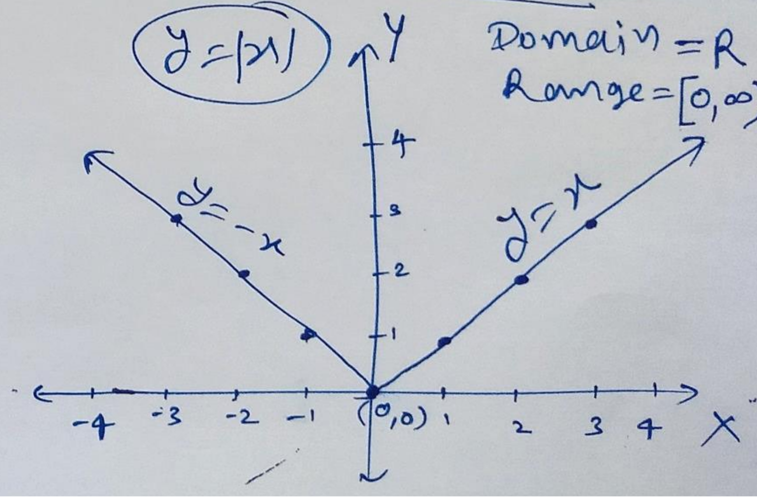
$y = -x$



$y = |x|$

x	y
0	0
1	1
2	2
-1	1
-2	2
-3	3
3	3

$y = |x|$



Domain = R

Range = $[0, \infty)$

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$-2 = (-1) \times 2$

(VI) Signum Function:

$f: R \to R$ defined by

$\overset{+}{\longleftarrow} \; - \; 0 \; + \; \longrightarrow$ ↑ no sign

$$y = f(x) = \operatorname{sgn}(x) = \begin{cases} +1 & , \text{ if } x > 0 \ (\text{Positive}) \\ 0 & , \text{ if } x = 0 \ (\text{zero}) \\ -1 & , \text{ if } x < 0 \ (\text{Negative}) \end{cases}$$

e.g. $f(2) = +1 = \operatorname{sgn}(2) \quad (\because 2 > 0)$

$f(-3) = -1 = \operatorname{sgn}(-3) \quad (\because -3 < 0)$

$\operatorname{sgn}(100) = +1$

$\operatorname{sgn}(0) = 0$

$\operatorname{sgn}(-20) = -1$

$\operatorname{sgn}(-\pi) = -1$

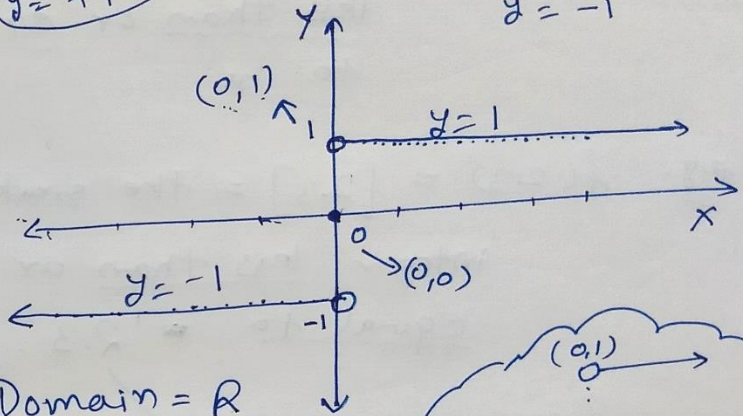
$\operatorname{sgn}(\pi) = +1$

Graph of Sgn(x):

x	0	1	2	3	4	...	-1	-2	-3	-4	...
$\operatorname{sgn}(x) = y$	0	1	1	1	1		-1	-1	-1	-1	

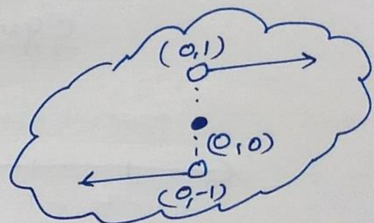
$x = 0.001$, $y = +1$

$x = -0.0001$, $y = -1$



Domain = R

Range = $\{1, 0, -1\}$



$$y = \operatorname{sgn}(x) = f(x) = \begin{cases} 1 & , x > 0 \\ 0 & , x = 0 \\ -1 & , x < 0 \end{cases} \quad x \neq 0$$

$$= \begin{cases} 0 & , x = 0 \\ \dfrac{|x|}{x} & , x \neq 0 \end{cases}$$

$x > 0$

$x = 3$

$\dfrac{|3|}{3} = \dfrac{3}{3} = 1$

$x < 0$

$x = -10$

$\dfrac{|-10|}{-10} = \dfrac{10}{-10} = -1$

$\dfrac{|x|}{x} = \dfrac{x}{|x|}$ ☆

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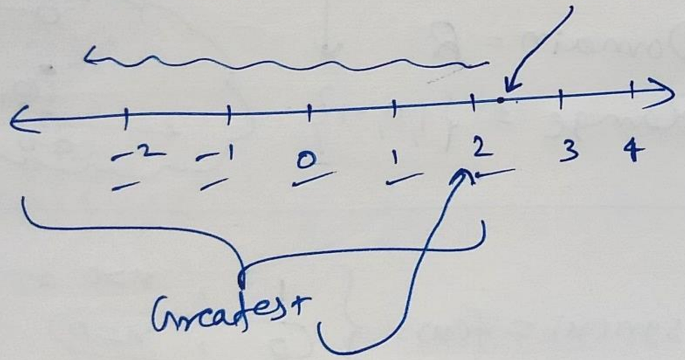
(VII) Greatest integer Function (GIF)

$f: R \to R$ defined by

$y = f(x) = [x]$ = the greatest integer less than or equal to 'x'

e.g. $f(2.3) = [2.3]$ = the greatest integer less than or equal to '2.3'.

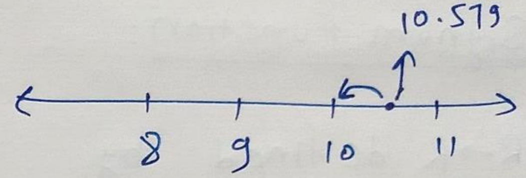


$f(2.3) = [2.3] = 2$

$[2.4] = 2$

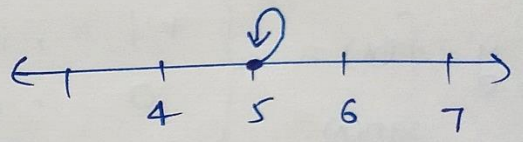
$[2.9] = 2$

e.g. $[10.579] = 10$

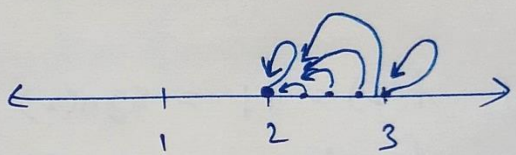


e.g. $[5] = 5$

integer

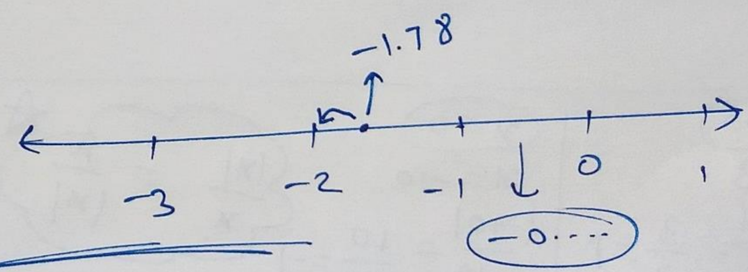


e.g.



$[2.99999] = 2$

e.g. $[-1.78] = -2$



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$[0] = 0$
$[0.1] = 0$
$[0.2] = 0$
$\vdots$
$[0.9] = 0$

$[1] = 1$
$[1.1] = 1$
$[1.2] = 1$
$\vdots$
$[1.999] = 1$

$[2] = 2$
$[2.1] = 2$
$[2.2] = 2$
$\vdots$
$[2.99] = 2$

$[3] = 3$
$[3.001] = 3$
$\vdots$
$[3.99] = 3$

$[4] = 4$
$[4.01] = 4$

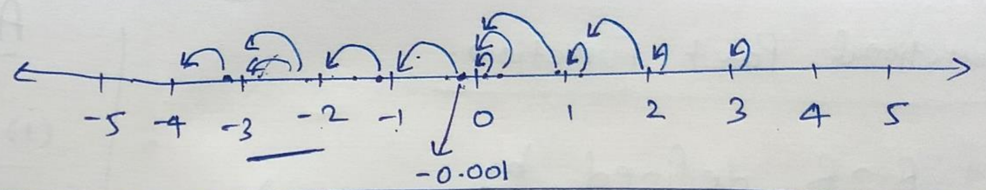
$[-0.001] = -1$
$[-0.2] = -1$
$[-0.9] = -1$
$[-1] = -1$

$[-1.1] = -2$
$[-1.5] = -2$
$[-1.9] = -2$
$[-2] = -2$

$[-2.001] = -3$
$[-2.2] = -3$
$[-2.3] = -3$
$[-2.99] = -3$
$[-3] = -3$

$[-3.1] = -4$

$[\pi] = 3$

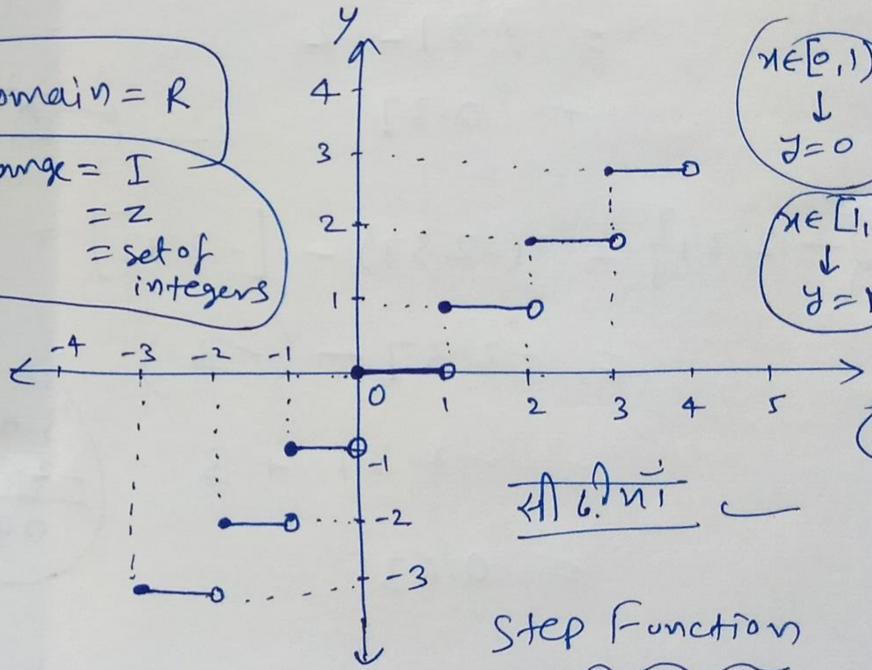


$[I] = I$
integer

$$y = f(x) = [x] = \begin{cases} -3, & x \in [-3,-2) \\ -2, & x \in [-2,-1) \\ -1, & x \in [-1,0) \\ 0, & x \in [0,1) \longrightarrow 0 \le x < 1 \\ 1, & x \in [1,2) \longrightarrow 1 \le x < 2 \\ 2, & x \in [2,3) \rightarrow 2 \le x < 3 \\ y = 3, & x \in [3,4) \end{cases}$$

Domain $= R$

Range $= I$
$= Z$
$=$ set of integers



$x \in [0,1)$
$\downarrow$
$y = 0$

$x \in [1,2)$
$\downarrow$
$y = 1$

$y = 0$

सीढ़ीनुमा

Step Function

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Fractional Part Function.

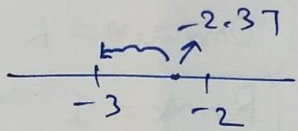
$f: R \to R$ defined by

$$y = f(x) = \boxed{\{x\} = x - \underbrace{[x]}_{\text{GIF}}}$$

$$\{2.37\} = 2.37 - \underbrace{[2.37]}_{\text{GIF}}$$

$$= 2.37 - 2$$

$$= 0.37$$



$$\{-2.37\} = (-2.37) - [-2.37]$$

$$= -2.37 - (-3)$$

$$= -2.37 + 3$$

$$= 0.63$$

$$\begin{array}{r} 3.00 \\ -2.37 \\ \hline 0.63 \end{array}$$

Algebra of Real Functions.

(i) Addition

$$(f+g)(x) = f(x) + g(x)$$

(ii) Subtraction =

$$f(x) - g(x) = (f-g)(x)$$

(iii) multiplication

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

(iv) Division

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad g(x) \neq 0$$

(v) Scalar muliplication

$$(\alpha f)(x) = \alpha . f(x)$$

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e.g. $f(x) = 2x+3 \quad x \in R$

$g(x) = x-7 \quad x \in R$

① $(f+g)(x) = f(x)+g(x) = 2x+3+x-7 = 3x-4 \quad , x \in R$

② $(f-g)(x) = f(x)-g(x) = (2x+3)-(x-7) = x+10 \quad , x \in R$

③ $(f \cdot g)(x) = (2x+3)(x-7) = 2x^2-11x-21 \quad , x \in R$

④ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)_{\neq 0}} = \frac{2x+3}{x-7 \neq 0}$ Domain $= x \in R-\{7\}$

⑤ $(7f)(x) = 7 \cdot f(x) = 7(2x+3) = 14x+21 \quad , x \in R$

$(8g)(x) = 8 \cdot g(x) = 8(x-7) = 8x-56 \quad , x \in R$

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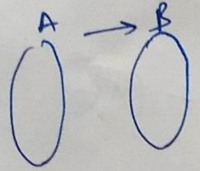
Exercise $\boxed{2.3}$ {all questions are related to functions}

① (i)

$\{(2,1), (5,1), (8,1), (11,1), (14,1), (17,1)\}$ ✓ Fn^n. ✓

Fn^n → first set should be Domain.
\+
No two ordered pairs have same first element ✓

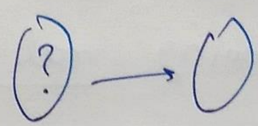


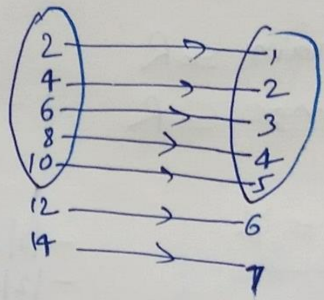
Domain = $\{2, 5, 8, 11, 14, 17\}$

Range = $\{1\}$

(ii)

$\{(2,1), (4,2), (6,3), (8,4), (10,5), (12,6), (14,7)\}$





Fn^n. ✓

Domain = $\{2, 4, 8, 6, 10, 12, 14\}$

Range = $\{1, 2, 3, 4, 5, 6, 7\}$ ✓

(iii) $\{\underline{(1,3)}, \underline{(1,5)}, (2,5)\}$

$(1,3), (1,5) \longrightarrow$ Same first element

∴ Not a function

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Exercise (2.3)

(2) Domain and Range of the following real function: $\rightarrow$

real function $\downarrow$

input $\rightarrow$ Domain $\subseteq R$

output $\rightarrow$ Range $\subseteq R$

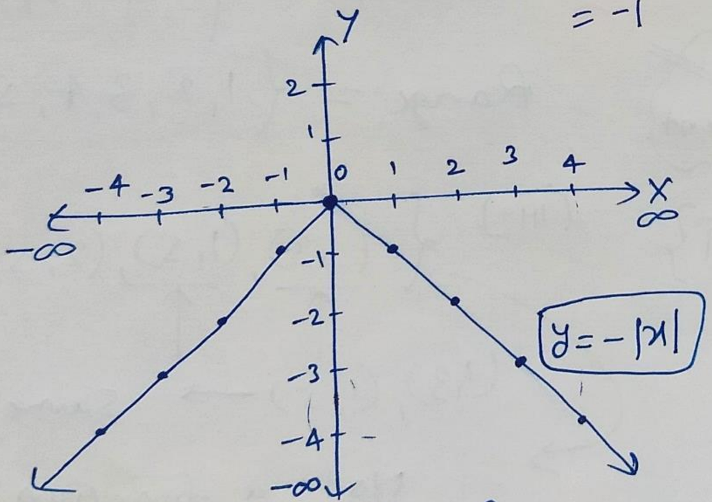
(i) $f(x) = -|x| = y$

$-|1| = -1$

$-|2| = -2$

$-|-1| = -(+1) = -1$

x	y
0	0
1	-1
2	-2
3	-3
4	-4
-1	-1
-2	-2
-3	-3
-4	-4
-5	-5



Domain $= (-\infty, \infty) = R$

(y) Range $= (-\infty, 0] \rightarrow \boxed{-\infty < y \leq 0}$

Second method

$f(x) = -|x|$

Domain $\rightarrow$ set of all possible inputs (which can be put in the function) x

$\sqrt{g(x)} \rightarrow g(x) \geq 0$

$\frac{g(x)}{h(x)} \rightarrow h(x) \neq 0$

Domain of $f(x) = -|x|$ is R.

Range: $y = f(x) = -|x|$

(Start with known expression)

$|x| \geq 0 \rightarrow 0 \geq -|x|$

$-|x| \leq 0$

$\Rightarrow y \leq 0 \Rightarrow -\infty < y \leq 0$

$y \in (-\infty, 0]$

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② (ii) $\boxed{f(x) = \sqrt{9-x^2}}$

Domain

(For Real Funⁿ.) ⇒ Coeff. of x^2 ⇓ (+)

$9 - x^2 \geq 0.$

$x = 10$
$\hookrightarrow \sqrt{9-100}$
$\sqrt{-91}$
↓
imaginary
(not real)

~~multipp~~ multiply ⊖ ↓ (sign of inequality changes)

$\Rightarrow x^2 - 9 \leq 0$

⇒ (factorisation)

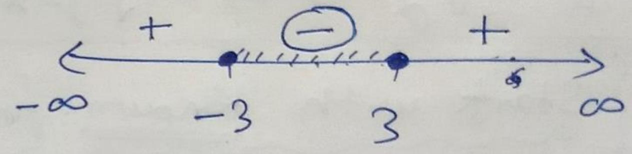
$\Rightarrow x^2 - 3^2 \leq 0$

$a^2 - b^2 = (a+b)(a-b)$

$\Rightarrow (x-3)(x+3) \leq 0$

Number line ↓ mark (roots)

roots
$x = 3$
$x = -3$



✓ (Sign allotment) → right most part = ⊕

then alternate sign while moving left.

✓ Solution → part.
(According to sign of inequality)

$(x-3)(x+3) \leq 0$
↑
less than or equal to → ⊖ve
→ कोने (corners) include.

$x \in [-3, 3]$
interval

$-3 \leq x \leq 3$
Domain

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Range of $y = f(x) = \sqrt{9-x^2}$

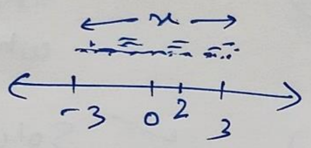
{ Start with Known function }

Universal truth for real x $\longrightarrow$ $x^2 \geq 0$

By Domain, we Know that

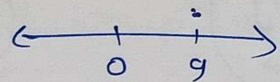
$x \in [-3, 3]$

$\Rightarrow -3 \leq x \leq 3$



$\Rightarrow$ Square

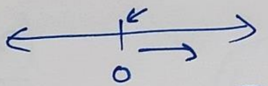
$\cancel{9 \leq x^2 \leq 9}$ X



$x^2 = 9$

$x = 2$

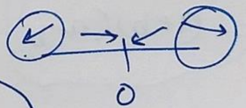
$x^2 = 4$



$x^2 = x \cdot x = \oplus$

$\quad\;\; - \;\; -$

$\quad\;\; + \;\; + \;\; \oplus$

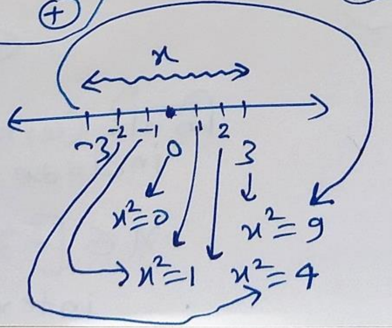


$-3 \leq x \leq 3$

$\quad\;\; \downarrow$

$\quad\;\; 0$

$0 \leq x^2 \leq 9$



$0 \leq x^2 \leq 9$

multiply with $\ominus$

$\Rightarrow 0 \geq -x^2 \geq -9$

$\Rightarrow -9 \leq -x^2 \leq 0$

$\Rightarrow 9-9 \leq 9-x^2 \leq 9+0$

$\Rightarrow 0 \leq 9-x^2 \leq 9$

Square root $\quad \pm$ ✓ X

$0 \leq \sqrt{9-x^2} \leq 3$

$\Rightarrow 0 \leq y \leq 3$

Range $= [0, 3]$

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Exercise 2.3 × Relations & Functions

③ $f(x) = 2x-3$.

(i) $f(0) = 2(0)-3 = 0-3 = -3$

(ii) $f(7) = 2(7)-3 = 14-3 = 11$

(iii) $f(-3) = 2(-3)-3 = -6-3 = -9$

④ input input

$$\underbrace{t(C)}_{\text{output} = F} = \frac{9C}{5} + 32$$

't' is a function to change celsius into Fahrenheit.

(i) $t(0) = \frac{9(0)}{5} + 32 = 32$ Fahrenheit

0° celsius

(ii) $t(28)$

$t(c) = \frac{9c}{5} + 32$

$t(28) = \frac{9 \times 28}{5} + 32$

$= \frac{252}{5} + \frac{32}{1} = \frac{252+160}{5}$

$= \frac{412}{5}$ Fah.

(iii) $t(-10) = \frac{9(-10)}{5} + 32$

$= 9(-2) + 32 = -18+32$

$= 14$

(iv) The value of c, when $t(c) = 212$

$\frac{9c}{5} + 32 = 212$

$\Rightarrow \frac{\cancel{9}c}{5} = \cancel{180}\ 20$

$\Rightarrow C = 20 \times 5 = 100$

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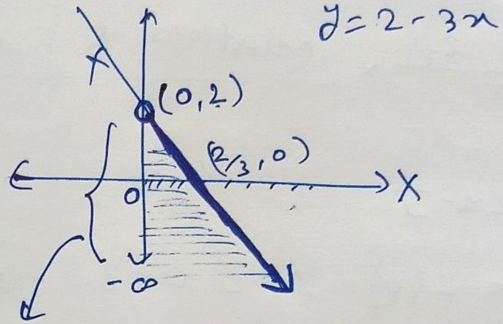
⑤ Find the range:

(i) $f(x) = 2-3x, \quad x \in R, \; x>0$

(Start with known function)

$y = \boxed{f(x) = 2-3x}$



Range $(-\infty, 2)$ Right side graph

we know that $x>0$ (Given)

$\Rightarrow$ multiply (-3) = -ve

$\downarrow$ sign change Inequality

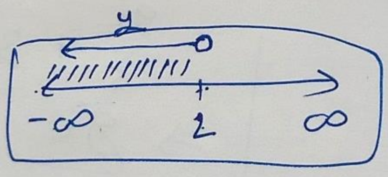
$\Rightarrow -3x < 0$

add 2

$\Rightarrow 2-3x < 2+0$

$\Rightarrow (2-3x) < 2$

$f(x) < 2$



$-\infty < y < 2$

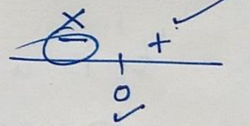
$y \in (-\infty, 2)$

(ii) $f(x) = x^2+2$, $x \in R$

x is any real no.

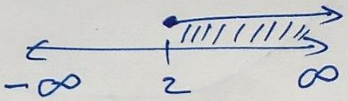
we know that

$x^2 \geq 0$



$\Rightarrow (x^2+2) \geq 0+2$

$y = f(x) \geq 2$



Range $= [2, \infty)$

(iii) $y = f(x) = x$, $x \in R$ (Identity Fn.)

$\Rightarrow x \in R$

$\Rightarrow y \in R$

y = output = Range.

$\rightarrow$ Range = R

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Miscellaneous Exercise 2

Chapter - 2

Relations and Functions

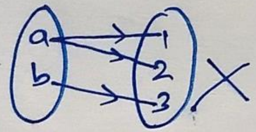
1.

$$f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$$

$$g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$$

$f(3) = 3^2$, (x^2)

$= 9$



$f(3) = 3 \times 3$, $(3x)$

$= 9$

$(3, 9)$

3 has unique image

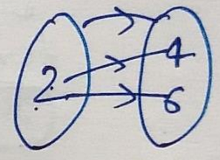
$g(2) = 2^2$ (according to x^2)

$= 4$

$g(2) = 3(2)$ (according to $3x$)

$= 6$

$(2,4)$, $(2,6)$



2 doesn't have unique image.

$\therefore$ f is a function but g is only a relation.

2. $f(x) = x^2$

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1)^2 - (1)^2}{1.1 - 1} = \frac{1.21 - 1}{0.1}$$

$$= \frac{0.21}{0.1} = 2.1$$

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(3) Find the domain of the function

$$f(x) = \frac{x^2+2x+1}{x^2-8x+12} = \frac{Nr.}{Dr. \neq 0}$$

Domain:

$\sqrt{f(x)} \longrightarrow f(x) \geq 0$

$\frac{f(x)}{g(x)} \longrightarrow g(x) \neq 0$

For Domain / Denom. $= x^2 - 8x + 12 \neq 0$

$\Rightarrow x^2 - 2x - 6x + 12 \neq 0$

$\Rightarrow x(x-2) - 6(x-2) \neq 0$

$\Rightarrow (x-2)(x-6) \neq 0$

$x \neq 2, \quad x \neq 6$

x can be anything but not 2 & 6.

$x \in R - \{2, 6\} =$ Domain

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Miscellaneous Exercise
Chapter (2)

(4) $f(x) = \sqrt{x-1}$

Domain Square root $\cancel{x-1 < 0}$

$x - 1 \geq 0$

$\Rightarrow x \geq 1$

$1 \quad \infty$

Domain $= [1, \infty)$ $\quad x \in [1, \infty)$

Range: $y = \sqrt{x-1}$ $\quad \sqrt{10-1}$

$y \geq x^2 \quad y \geq + x$

x	1	2	3	4	5	10
y	0	1	$\sqrt{2}$	$\sqrt{3}$	2	3

$y \in [0, \infty)$

$y = \sqrt{1-x}$

Range: Start with Known Function.

Due to Domain $x - 1 \geq 0$

$\sqrt{x-1} \geq 0 \Rightarrow y \geq 0$

(5) $f(x) = |x-1|$

Domain

neither $\sqrt{\dots}$ nor $\frac{Nr}{Dr}$

$x = (-)$

$x = (+)$

$x = 0$

we can put all real numbers.

Domain $= x \in R$

Range. $y = |x-1|$ $\quad |x| \geq 0$

we Know that

$|x-1| \geq 0$

$y \geq 0$

$\Rightarrow y \in [0, \infty)$

x	y
0	1
1	0
2	1
3	2
-1	2
-2	3
-3	4

0, (+)ve

Range $= [0, \infty)$

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(6)

$f = \left\{ \left(x, \frac{x^2}{1+x^2}\right) : x \in R \right\}$

$f = \{ \underbrace{(x, y)}_{\text{Ordered pair}} : x \in R,$ relation b/w x & y $\}$

f^n

$f = \left\{ (x,y) : x \in R, \; y = \frac{x^2}{1+x^2} \right\}$

$f(x) = y = \frac{x^2}{1+x^2}, \quad x \in R$

I - method Range

↓

Start with known f^n.

$y = \frac{x^2}{1+x^2}$

$y = \frac{x^2+1-1}{\boxed{1+x^2}} = \frac{x^2+1}{x^2+1} - \frac{1}{1+x^2}$

$y = 1 - \frac{1}{1+x^2}$ Single Place of x

Start with Known F^n.

we know that $x^2 \geq 0$

$\Rightarrow 1+x^2 \geq 1+0$

$\Rightarrow$

$\infty > 1+x^2 \geq 1$

$\frac{1}{\infty} = 0$

Reciprocal. (Sign of In. → change)

$\frac{1}{1} = 1$

$\Rightarrow 0 < \frac{1}{1+x^2} \leq 1$

$\Rightarrow 0 > \frac{-1}{1+x^2} \geq -1$

$\Rightarrow 1 > \boxed{1 - \frac{1}{1+x^2}} \geq 1-1$

$\Rightarrow 1 > y \geq 0$

$\Rightarrow 0 \leq y < 1$

$y \in [0, 1)$

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II - method. Range = ? $x \in R$

Represent 'x' in terms of 'y'

$$y = \frac{x^2}{1+x^2}$$ ← Now y is represented in terms of x

$\Rightarrow y + y.x^2 = x^2$

$\Rightarrow y = -y.x^2 + x^2$

$\Rightarrow x^2.(-y+1) = y$

$\Rightarrow x^2 = \frac{y}{1-y}$

$\Rightarrow x = \pm\sqrt{\frac{y}{1-y}}$ ✓

Given $x \in R$

x is Real. ✓

$\sqrt{-}$

$\sqrt{\frac{y}{1-y}} \rightarrow$ Real ✓

$\frac{y}{1-y} \geq 0 \longrightarrow \frac{y}{-(y-1)} \geq 0$

$\Rightarrow \boxed{\frac{y}{y-1} \leq 0}$

$y = 0 \rightarrow \boxed{y=0}$

$y-1=0 \rightarrow y=1$

Always Exclude

+ ⊖ +

0 1

$y \in [0, 1)$

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Misc. Chapter 2

7 $f, g : R \to R$ | $f(x) = x+1$, $g(x) = 2x-3$

$f+g$

$\downarrow$

$(f+g)(x) = f(x) + g(x)$

$= (x+1) + (2x-3)$

$= 3x - 2$ ✓ $x \in R$

$f-g \to (f-g)(x) = f(x) - g(x)$

$= (x+1) - (2x-3)$

$= -x + 4$ ✓ $x \in R$

$\frac{f}{g} \to \frac{f(x)}{g(x)} = \frac{x+1}{2x-3}$ ✓

Dr. $\neq 0$

$2x-3 \neq 0$

$x \neq 3/2$

| $x \in R - \left\{\frac{3}{2}\right\}$

8 $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ $\to (x, y)$

$\boxed{Z \text{ to } Z}$

$y = f(x) = ax+b$

$(1,1)$

Domain $= \{1, 2, 0, -1\}$

Range $= \{1, 3, -1, -3\}$

$1 = a \times 1 + b$

$\Rightarrow \boxed{a+b=1}$ — (1)

$y = ax+b$

$(0,-1) \equiv (x, y)$

$\Rightarrow -1 = a(0) + b$

$\Rightarrow -1 = 0 + b$

$\Rightarrow \boxed{b=-1}$ — (2)

$b = -1$ ✓

By eq^n. (1)

$a + (-1) = 1$

$\Rightarrow a = 2$ ✓

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9) R relation from N to N

$R = \{(a, b),\ a, b \in N,\ a = b^2\} \to (a, b) \in R$

(i) $(a, a) \in R$, for all $a \in N$

$\begin{cases} (1,1) \in R \\ 1 = 1^2 \checkmark \end{cases}$ $a = 1, 2, 3, 4, \ldots$

$\begin{cases} (2,2) \in R \\ 2 = 2^2 \times \end{cases}$ False

when $(a, b) \in R \Rightarrow a = b^2$

$(a, a) \in R \Rightarrow a = a^2$ False

$\Rightarrow a^2 - a = 0$

$\Rightarrow a(a-1) = 0$

$a = 0$ ✗ $0 \notin N$ | $a = 1 \in N$ only one value of a, not for all $a \in N$

(ii) $(a, b) \in R$, implies $(b, a) \in R$

$a = b^2 \not\longrightarrow b = a^2$

let $\left.\begin{array}{l} a = 9 \\ b = 3 \end{array}\right\}$ $9 = 3^2$ | (b, a)

$(a, b) \equiv (9, 3) \in R \longrightarrow (3, 9) \in R$???

$3 = 9^2$

False

$\Rightarrow 3 = 81$ ✗

(iii) if $(a, b) \in R$, $(b, c) \in R$ implies $(a, c) \in R$

let

$a = (b)^2$ $b = c^2$ $\Longrightarrow$ Prove $\boxed{a = c^2}$???

$a = (c^2)^2$

$\Rightarrow \boxed{a = c^4} \checkmark$

False

Misc. Ex. 2

(10)

$A = \{1, 2, 3, 4\}$

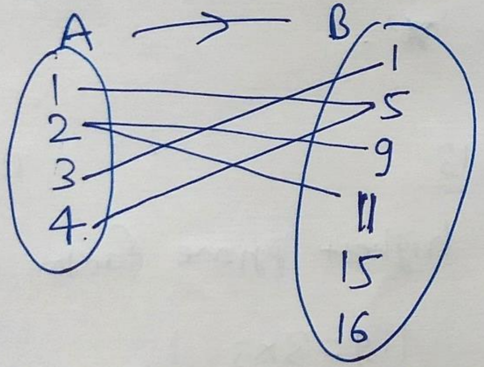
$B = \{1, 5, 9, 11, 15, 16\}$

$f = \{(1,5), (2,9), (3,1), (4,5), (2,11)\}$

(i) f is a relation from A to B

(ii) f is a function from A to B.

(i)



True.

f is a relation from A to B.

Domain $= \{1, 2, 3, 4\} = A$

Range $= \{1, 5, 9, 11\} \subset B$

(ii)

function

(i) 1^{st} set A = Domain ✓

(ii) each element of set A must has unique image in set B.

2 has 2 images.

$\Rightarrow f$ is not a function

(11) ~~f is subset of Z × Z~~

~~f = {~~

(11) f is subset of $Z \times Z$

~~$f = \{(ab, a+b)\}$~~

$f = \{(ab, a+b) : a, b \in Z\}$

$f = \{(x, y) : \_\_\_\_\_\_\}$

Is f function from Z to Z?

$(x, y) \in f$

$10 = x = ab$

$y = a+b$

$a, b \in Z$

let $a = 10, b = 1$

$a \times b = 10 \times 1 = 10 = x$

$y = a+b = 10+1 = 11$

$(x, y) \in f$

$(10, 11) \in f \longleftrightarrow (10, 7) \in f$

let $a = 2, b = 5 \longrightarrow x = ab = 2 \times 5 = 10$

$y = a+b = 2+5 = 7$

$\because$ '10' has 2 images.

$\Rightarrow f$ is not a function.

---

(12) $A = \{9, 10, 11, 12, 13\}$

$f : A \to N$

$f(n)$ = the highest prime factor of n.

$f(x)$ = the highest prime factor of 'x'.

input.

A

$x = 9, 10, 11, 12, 13$

$10 = 2 \times 5$

$f(10)$ = the highest prime factor of '10'

$= 5$

$f(9) = 3$

$f(11) = 11$

$f(12) = 3$

$f(13) = 13$

$9 = 3 \times 3$

$11 = 1 \times 11$

$12 = 2 \times 3 \times 2$

$13 = 1 \times 13$

Range $= \{3, 5, 11, 13\}$

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